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ORIGINAL ARTICLE

New Bayesian Estimators for Randomized Response Technique

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Abstract

This paper proposes new Bayesian estimators of the population proportion of a sensitive attribute when life data were collected through the administration of questionnaires on abortion on 300 matured women in some selected hospitals in Akure, Ondo State, Nigeria. Assuming both the Kumaraswamy (KUMA) and the generalised (GLS) beta distributions as alternative beta priors, efficiency of the proposed Bayesian estimators was established for a wide interval of the values of the population proportion (π). We observed that for small, medium as well as large sample sizes, the developed Bayesian estimators were better in capturing responses from respondents than the conventional simple beta estimator proposed by Hussain and Shabbir (2009a) as π approaches one.

Keywords: Alternative beta priors; sensitive attribute; mean square error; absolute bias; efficiency.

Introduction

Direct questioning about a sensitive attribute such as induced abortion, use of drug, tax evasion, etc. in a human population survey is a strenuous exercise. A survey statistician may receive wrong responses from the survey respondents when he/she uses direct questioning technique. Due to many reasons, information about incidence of sensitive attributes in the population becomes essential. Warner (1965) was the first to put forward a method of survey to collect information in relation to sensitive attributes by ensuring privacy and anonymity to the respondents. To date, numerous developments and improvements on Warner's Randomized Response Technique have been developed by many researchers. Greenberg et al. (1969), Mangat and Singh (1990), Mangat (1994), Singh et al. (1998), Christofides (2003), Kim and Warde (2004), Adebola and Adepetun (2011), Adebola and Adepetun (2012), Adepetun and Adebola (2014) are some of the many to be cited. In some situations, prior information about the unknown parameter may be available and can be combined with the sample information for the estimation of that unknown parameter. This is known as the Bayesian approach of estimation. Work done by researchers on Bayesian analysis of Randomized response models are not very elaborate, however, attempts have been made on the Bayesian analysis of Randomized response techniques. Winkler and Franklin (1979), Pitz (1980), Spurrier and Padgett (1980), O'Hagan (1987), Oh (1994), Migon and Tachibana

(1997), Unnikrishnan and Kunte (1999), Bar-Lev and Bobovich (2003), Barabesi and Marcheselli (2006), Kim et al. (2006), Hussain and Shabbir (2009a, 2009b), Hussain and Shabbir (2012), Adepetun and Adewara (2014), are the major references on the Bayesian analysis of the Randomized Response Techniques. In this study, we propose new Bayesian estimators of the population proportion of respondents who possess ignoble attribute in a simple random sample of size n assuming different beta priors other than the conventional simple beta prior found in the literature..

Materials and Methods

In Bayesian Analysis, the prior information about the unknown parameter of the population is combined with the sample information for the determination of that unknown parameter. Notable authors like Winkler and Franklin (1979), O'Hagan (1987), Kim et al. (2006), Hussain and Shabbir (2012) etc. have provided Bayesian analysis to some randomized response techniques in the literature using simple beta prior.

In this study, we presented both the conventional and the alternative beta priors for the randomized response technique. Similarly, we assume numerical values for the parameters in the priors. In the case of simple beta prior, we assume $a>1$, $b>1$, $a\neq b$, $c=1$. For Kumaraswamy prior, we assume $a=1$, $b>1$, $c>1$, $b\neq c$. For the generalised beta prior, we assume $a>1$, $b>1$, $c>1$, $a\neq b\neq c$ respectively. Consequently, the conventional simple beta estimator along with the proposed estimators assuming Kumaraswamy and the generalised beta priors were derived and computed from their respective posterior distributions using R statistical software respectively.

The tables showing absolute bias, mean square errors and relative efficiencies were displayed for comparison. Consequently, graphs of the mean square errors, absolute bias of the estimators were plotted against assigned values of π in the range $0<\pi<1$ using selected sample sizes 25,100, and 250 respectively. Life data obtained from administered survey questionnaires on induced abortion among 300 women in Akure, Ondo State were also used to establish the efficiency of the proposed estimators in capturing responses from respondents with respect to stigmatized attribute..

The Existing Bayesian Technique of Estimation

Hussain and Shabbir (2009a) in their referred paper presented a Bayesian estimation to the Randomized Response Technique put forward by Hussain and Shabbir (2007) using a simple beta prior distribution to estimate the population proportion of respondents possessing sensitive attribute.

Let the simple beta prior be defined as follows

$$\mathrm{f}(\pi)=\frac{1}{\beta(a,b)}\pi^{a-1}(1-\pi)^{b-1};\quad 0<\pi<1 \tag{1}$$

where (a,b) are the shape parameters of the distribution and π is the population proportion of respondents possessing the sensitive attribute.

Let $x=\sum x_i$ be the total number of the women who have committed abortion for a particular sample of size n selected from the population with simple random sampling with replacement sampling. Then the conditional distribution of X given π was

$$f(X|\pi)=\frac{n!}{x!(n-x)!}\phi^{x}(1-\phi)^{n-x} \tag{2}$$

where ϕ is the probability of "yes response" to the sensitive attribute which was defined as

$$\phi = \frac{\alpha}{\alpha+\beta}\left(P_1\pi + (1-P_1)(1-\pi)\right) + \frac{\beta}{\alpha+\beta}\left(P_2\pi + (1-P_2)(1-\pi)\right) \tag{3}$$

where P_1 is the predetermined probability of "yes" response to the sensitive attribute and (α, β) are non-zero constants such that $P_1 + P_2 = 1$ respectively.

Thus,

$$f(X|\pi) = \binom{n}{x}\left[\frac{\pi\left(((2P_1-1)(\alpha-\beta)) + (\beta P_1 + \alpha P_2)\right)}{\alpha+\beta}\right]^x \left[1 - \frac{\pi\left(((2P_1-1)(\alpha-\beta)) + (\beta P_1 + \alpha P_2)\right)}{\alpha+\beta}\right]^{n-x} \tag{4}$$

On simplification, it led to

$$f(X|\pi) = \binom{n}{x}\left[\frac{((2P_1-1)(\alpha-\beta))}{\alpha+\beta}\right](\pi+F)^x(1-\pi+H)^{n-x}$$

where

$$F = \frac{\beta P_1 + \alpha P_2}{(2P_1-1)(\alpha-\beta)}; \quad H = \frac{3P_1(\beta-\alpha)+3\alpha}{(2P_1-1)(\alpha-\beta)}$$

Setting

$$A = \binom{n}{x}\left[\frac{((2P_1-1)(\alpha-\beta))}{\alpha+\beta}\right]^n$$

$$f(X|\pi) = A\sum_{i=0}\sum_{j=0}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^i(1-\pi)^j$$

for $x = 0, 1, 2, \ldots, n$

Thus, the joint probability density functions (pdf) of X and π was

$$f(X,\pi) = D\sum_{i=0}\sum_{j=0}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^i\pi^{a-1}(1-\pi)(1-\pi)^{b-1} \tag{5}$$

where

$$D = \frac{\binom{n}{x}}{\beta(a,b)}\left[\frac{((2P_1-1)(\alpha-\beta))}{\alpha+\beta}\right]^n$$

Now the marginal distribution of *X* can be obtained by integrating the joint distribution of *X* and π with respect to π. Thus, the marginal distribution of *X* was given as

$$f(X)=\int_0^1 f(X,\pi)d\pi = D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\int_0^1 \pi^{a-1+i}(1-\pi)^{b-1+j}d\pi$$

$$f(x)=D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\beta(a+i,b+j) \quad (6)$$

The posterior distribution of π given *X* was defined as $f(\pi|x)=\frac{f(x,\pi)}{f(x)}$ (7)

$$f(\pi|x)=\frac{D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^{i}\pi^{a-1}(1-\pi)^{j}(1-\pi)^{b-1}}{D\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\beta(a+i,b+j)}$$

$$f(\pi|x)=\frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^{a-1+i}(1-\pi)^{b-1+j}}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\beta(a+i,b+j)} \quad (8)$$

The Bayes estimator of π which is the posterior mean of (8) was given as

$$\hat{\pi}_{SH}=\int_0^1 \pi f(\pi|x)d\pi=\frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\int_0^1 \pi^{a+1}(1-\pi)^{b+j-1}d\pi}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\beta(a+i,b+j)}$$

$$=\frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\beta(a+i+1,b+j)}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\beta(a+i,j+1)} \quad (9)$$

The Bias of $\hat{\pi}_{SH}$ and its mean square error were given as $B(\hat{\pi}_{SH})=E(\hat{\pi}_{SH})-\pi$ (10)

$$MSE(\hat{\pi}_{SH})=\sum_{x=0}^{n}(\hat{\pi}_{SH}-\pi)^2\binom{n}{x}\phi^{x}(1-\phi)^{n-x} \quad (11)$$

The Proposed Bayesian Techniques of Estimation

In this section, we develop alternative Bayesian estimators to Hussain and Shabbir (2009a) Randomized Response Technique using both the Kumaraswamy (KUMA) and the generalised (GLS) beta prior distributions as our alternative beta prior distributions in addition to the simple beta prior distribution used by Hussain and Shabbir (2009a).

Estimation of π Using Kumaraswamy Prior

The Kumaraswamy prior distribution of π is given as $f(\pi) = bc\pi^{c-1}(1-\pi^c)^{b-1}; b,c > 0$ (12)

Using the Kumaraswamy prior in (12), the joint probability density function of X and π is derived as

$$f(x,\pi) = bcE\sum_{i=0}^{x}\sum_{j=0}^{n-x}\binom{x}{i}\binom{n-x}{j}F^{x-i}H^{n-x-j}\pi^{i}(1-\pi)^{j}(1-\pi^{c})^{b-1}\pi^{c-1} \tag{13}$$

Where $E = \binom{n}{x}\left[\frac{((2P_1-1)(\alpha-\beta))}{\alpha+\beta}\right]^n$

The marginal probability density function (pdf) of X can be obtained as

$$f(X) = \int_0^1 f(X,\pi)d\pi \tag{14}$$

$$f = bcE\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}\int_0^1(1-\pi)^{j}\pi^{ck+i+c-1}d\pi$$

$$= abE\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}\beta(ck+c+i, j+1) \tag{15}$$

Similarly, the posterior distribution as usual is obtained as follows $f(\pi|x) = \frac{f(x,\pi)}{f(x)}$

$$= \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}(1-\pi)^{j}\pi^{ck+i+c-1}}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}\beta(ck+c+i, j+1)} \tag{16}$$

Under the Square error loss, we proceed to obtain the posterior mean which is the Bayes estimator as follows $\hat{\pi}_{KH} = \int_0^1 \pi f(\pi|x)d\pi$ (17)

Considering the fact that

$$\int_0^1 \pi(1-\pi)^{j}\pi^{ck+i+c-1}d\pi = \int_0^1 \pi^{ck+i+c-1}(1-\pi)^{j}d\pi = \beta(ck+i+c+1, j+1)$$

Therefore,

$$\hat{\pi}_{KH} = \frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}\beta(ck+i+c+1, j+1)}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}\beta(ck+c+i, j+1)} \tag{18}$$

As a result, the Bias of $\hat{\pi}_{KH}$ as well as its mean square error is also given as
$$B(\hat{\pi}_{KH}) = E(\hat{\pi}_{KH}) - \pi \tag{19}$$

$$MSE(\hat{\pi}_{KH}) = \sum_{x=0}^{n} (\hat{\pi}_{KH}) - \pi \binom{n}{x} \phi^{x} (1-\phi)^{n-x} \tag{20}$$

Estimation of π Using the Generalised Beta Prior

The generalised beta prior is defined as

$$f(\pi) = \frac{c}{\beta(a,b)} \pi^{ac-1} (1-\pi^{c})^{b-1}; a,b,c > 0 \tag{21}$$

Where a,b,c are the shape parameters of the prior distribution as given in equation (21)

From binomial series expansion, we know that

$$(1-\pi^{c})^{b-1} = \sum_{k=0}^{b-1} (-1)^{k} \binom{b-1}{k} (\pi^{c})^{k}$$

Consequently

$$f(\pi) = \frac{c}{\beta(a,b)} \sum_{k=0}^{b-1} (-1)^{k} \binom{b-1}{k} \pi^{c(k+a)-1}$$

As a result, the joint density function of π and X with the generalized beta prior is

$$f(X,\pi) = G \sum_{i=0}^{x} \sum_{j=0}^{n-x} \sum_{k=0}^{b-1} (-1)^{k} \binom{x}{i} \binom{n-x}{j} \binom{b-1}{k} F^{x-i} H^{n-x-j} (1-\pi)^{j} \pi^{c(a+k)+i-1} \tag{22}$$

where

$$G = \frac{c}{\beta(a,b)} \binom{n}{x} \left[\frac{((2P_1 - 1)(\alpha - \beta))}{\alpha + \beta} \right]$$

The marginal probability density function (pdf) of X can then be obtained from (22) as
$$f(X) = \int_{0}^{1} f(X,\pi) d\pi \tag{23}$$

$$= G \sum_{i=0}^{x} \sum_{j=0}^{n-x} \sum_{k=0}^{b-1} (-1)^{k} \binom{x}{i} \binom{n-x}{j} \binom{b-1}{k} F^{x-i} H^{n-x-j} \beta(c(k+a)+i, j+1) \tag{24}$$

Similarly, we obtained the posterior distribution of π given X as

$$f(\pi|x)=\frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}(1-\pi)^j\pi^{c(a+k)+i-1}}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}\beta(c(k+a)+i,j+1)} \tag{25}$$

In the same manner, under the square error loss, the posterior mean which is otherwise known as the Bayes estimator is given as

$$\hat{\pi}_{GH}=\frac{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}\beta(c(k+a)+i+1,j+1)}{\sum_{i=0}^{x}\sum_{j=0}^{n-x}\sum_{k=0}^{b-1}(-1)^k\binom{x}{i}\binom{n-x}{j}\binom{b-1}{k}F^{x-i}H^{n-x-j}\beta(c(k+a)+i,j+1)} \tag{26}$$

The Bias of $\hat{\pi}_{GH}$ and its mean square error are respectively given as

$$B(\hat{\pi}_{GH})=E(\hat{\pi}_{GH})-\pi \tag{27}$$

$$MSE(\hat{\pi}_{GH})=\sum_{x=0}^{n}(\hat{\pi}_{GH}-\pi)^2\binom{n}{x}\phi^x(1-\phi)^{n-x} \tag{28}$$

Remarks: Equation (9) is the conventional Bayesian estimator of the population proportion (π) of the respondents who have committed the sensitive attribute proposed by Hussain and Shabbir (2009a) while equations (18) and (26) are the newly proposed Bayesian estimators of the population proportion (π) of the respondents who have committed the sensitive attribute (the sensitive attribute in this case is abortion).

Relative Efficiency (RE) of the proposed estimator = MSE of the proposed estimator/MSE of the conventional estimator *100%.

Application

In this section, the proposed Bayesian estimators were applied to life data obtained from the administered survey questionnaires on induced abortion under the same values of parameters in the estimators using sample sizes 25, 100 and 250 respectively and compare the results with the conventional simple beta estimator proposed by Hussain and Shabbir (2009a). We overcame the associated computational problems by writing computer programs using available statistical software. Few results in tables and figures were presented to reduce spaces as follows:

Table 1. Mean square errors and Relative Efficiency for Hussain and Shabbir (2007) RRT at $n = 25, x = 11, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

π	MSE BETA	MSE KUMA	MSE GLS	RE KUMA	RE GLS
0.1	4.225819E-10	1.017346E-08	1.607482E-08	2411.347500	3806.146600
0.2	2.767500E-12	6.719137E-09	1.164096E-08	242599.2780	418772.5632
0.3	2.968604E-10	3.978724E-09	7.921015E-09	1340.067300	2666.666700
0.4	1.304861E-09	1.952218E-09	4.914973E-09	150.0000000	377.6923000
0.5	3.026768E-09	6.396192E-10	2.622838E-09	21.12210000	86.46860000
0.6	5.462583E-09	4.092771E-11	1.044610E-09	0.749100000	19.04760000
0.7	8.612305E-09	1.561435E-10	1.802902E-10	1.811800000	2.090600000
0.8	1.247593E-08	9.852666E-10	2.987716E-11	7.880000000	0.239200000
0.9	1.705347E-08	2.528297E-09	5.933715E-10	14.79530000	3.467800000

Table 2. Absolute Bias for Hussain and Shabbir (2007) RRT at $n = 25, x = 11, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

π	|BIAS|BETA	|BIAS|KUMA	|BIAS|GLS
0.1	0.10880517	0.53386124	0.67106898
0.2	0.00880517	0.43386124	0.57106898
0.3	0.09119483	0.33386124	0.47106898
0.4	0.19119483	0.23386124	0.37106898
0.5	0.29119483	0.13386124	0.27106898
0.6	0.39119483	0.03386124	0.17106898
0.7	0.49119483	0.06613876	0.07106898
0.8	0.59119483	0.16613876	0.02893102
0.9	0.69119483	0.26613876	0.12893102

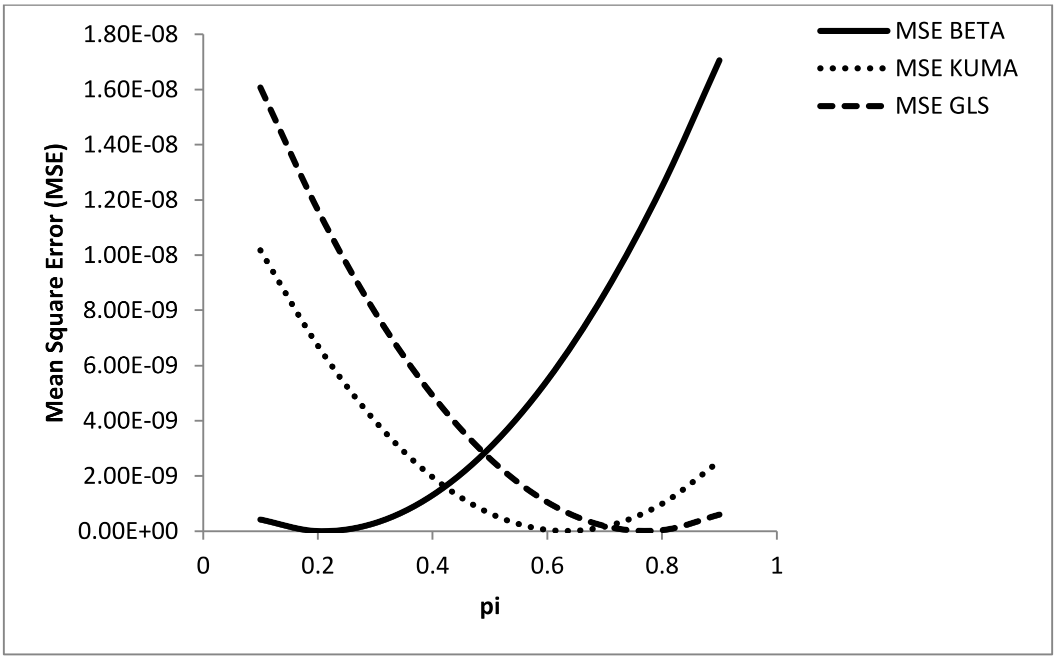

Figure 1. Mean square errors for Hussain and Shabbir (2007) RRT at $n = 25, x = 11, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

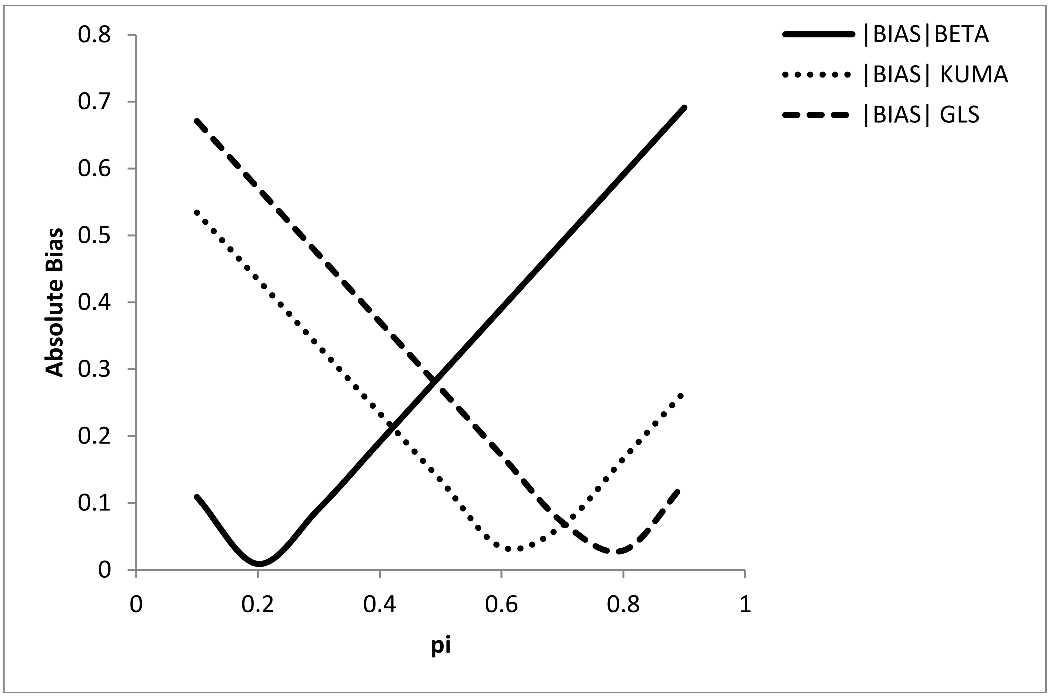

Figure 2. Absolute Bias for Hussain and Shabbir (2007) RRT at $n = 25, x = 11, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

Comment: When $n = 25, P_1 = 0.1$, the conventional simple beta estimator is better than the proposed estimators when π lies within the range $0.1 \leq \pi \leq 0.4$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.4 \leq \pi \leq 1$ However, the proposed generalised beta estimator is the best in capturing responses from respondents when π lies within the range $0.7 \leq \pi \leq 1$ respectively.

Table 3. Mean square errors and Relative Efficiency for Hussain and Shabbir (2007) RRT at $n = 25, x = 11, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.8$

π	MSE BETA	MSE KUMA	MSE GLS	RE KUMA	RE GLS
0.1	3.980527E-09	1.327131E-08	1.886529E-08	334.1709000	474.8744000
0.2	1.953481E-09	9.275222E-09	1.403224E-08	475.8974000	717.9487000
0.3	6.403423E-10	5.993041E-09	9.913099E-09	935.9375000	1548.437500
0.4	4.111079E-11	3.424768E-09	6.507863E-09	8321.167900	15839.41610
0.5	1.557865E-10	1.570402E-09	3.816535E-09	1006.410300	2448.717900
0.6	9.843696E-10	4.299438E-10	1.839114E-09	43.69920000	186.9919000
0.7	2.526860E-09	3.392482E-12	5.756007E-10	0.134000000	22.76680000
0.8	4.783257E-09	2.907485E-10	2.599432E-11	6.087900000	0.543900000
0.9	7.753562E-09	1.292012E-09	1.902953E-10	16.64520000	2.451600000

Table 4. Absolute Bias for Hussain and Shabbir (2007) RRT at
$n = 25, x = 11, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.8$

π	|BIAS|BETA	|BIAS|KUMA	|BIAS|GLS
0.1	0.33393689	0.60974884	0.72698568
0.2	0.23393689	0.50974884	0.62698568
0.3	0.13393689	0.40974884	0.52698568
0.4	0.03393689	0.30974884	0.42698568
0.5	0.06606311	0.20974884	0.32698568
0.6	0.16606311	0.10974884	0.22698568
0.7	0.26606311	0.00974884	0.12698568
0.8	0.36606311	0.09025116	0.02698568
0.9	0.46606311	0.19025116	0.07301432

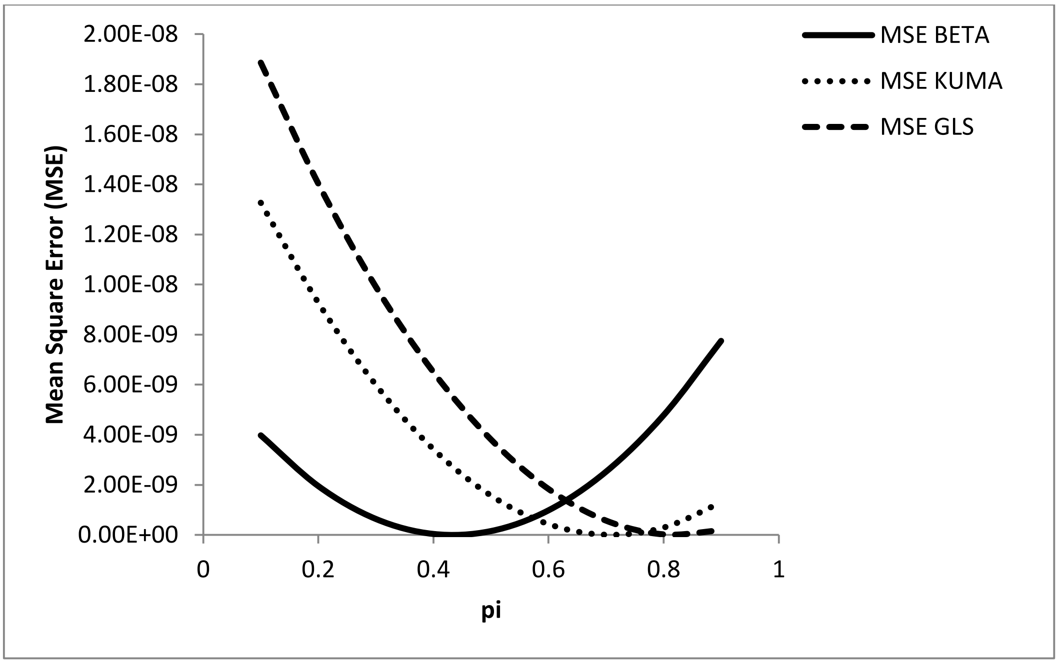

Figure 3. Mean square errors for Hussain and Shabbir (2007) RRT at
$n = 25, x = 11, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.8$

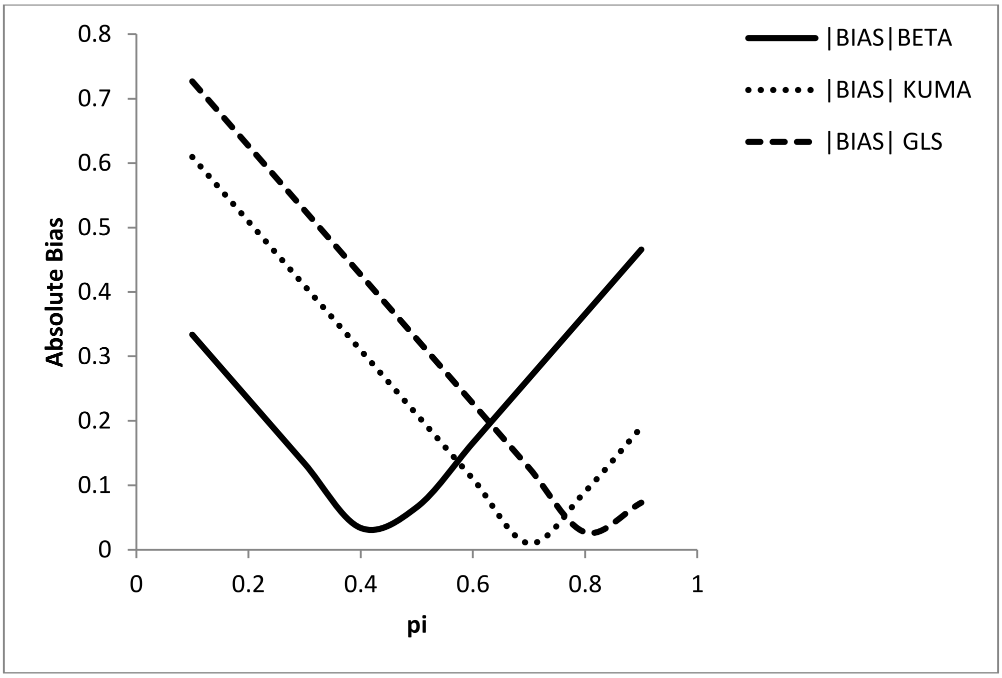

Figure 4. Absolute Bias for Hussain and Shabbir (2007) RRT at $n=25, x=11, \alpha=1, \beta=10, P_1=0.1, P_2=0.8$

Comment: When $n=25, P_1=0.2$, the conventional simple beta estimator is better than the proposed estimators when π lies within the range $0.1 \le \pi \le 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.5 \le \pi \le 1$. However, the proposed generalised beta estimator is the best in capturing responses from respondents when π lies within the range $0.7 \le \pi \le 1$ respectively.

Table 5. Mean square errors and Relative Efficiency for Hussain and Shabbir (2007) RRT at $n=100, x=43, \alpha=1, \beta=10, P_1=0.1, P_2=0.9$

π	MSE BETA	MSE KUMA	MSE GLS	RE KUMA	RE GLS
0.1	7.321979E-33	6.766773E-34	5.777443E-33	924.8634000	78.96170000
0.2	5.326008E-32	2.931831E-32	4.789110E-33	54.97190000	8.986900000
0.3	1.413718E-31	1.001335E-31	4.597435E-32	70.92200000	32.62410000
0.4	2.716570E-31	2.131223E-31	1.293332E-31	78.30880000	47.42650000
0.5	4.441158E-31	3.682846E-31	2.548655E-31	82.88290000	57.43240000
0.6	6.587482E-31	5.656206E-31	4.225715E-31	85.88770000	64.18820000
0.7	9.155541E-31	8.051300E-31	6.324510E-31	87.88210000	68.99560000
0.8	1.214534E-30	1.086813E-30	8.845041E-31	90.08260000	73.14050000
0.9	1.555687E-30	1.410670E-30	1.178731E-30	90.38460000	75.64100000

Table 6. Absolute Bias for Hussain and Shabbir (2007) RRT at
$n = 100, x = 43, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

π	|BIAS|BETA	|BIAS|KUMA	|BIAS|GLS
0.1	0.05892628	0.01791371	0.05234349
0.2	0.15892628	0.11791371	0.04765651
0.3	0.25892628	0.21791371	0.14765651
0.4	0.35892628	0.31791371	0.24765651
0.5	0.45892628	0.41791371	0.34765651
0.6	0.55892628	0.51791371	0.44765651
0.7	0.65892628	0.61791371	0.54765651
0.8	0.75892628	0.71791371	0.64765651
0.9	0.85892628	0.81791371	0.74765651

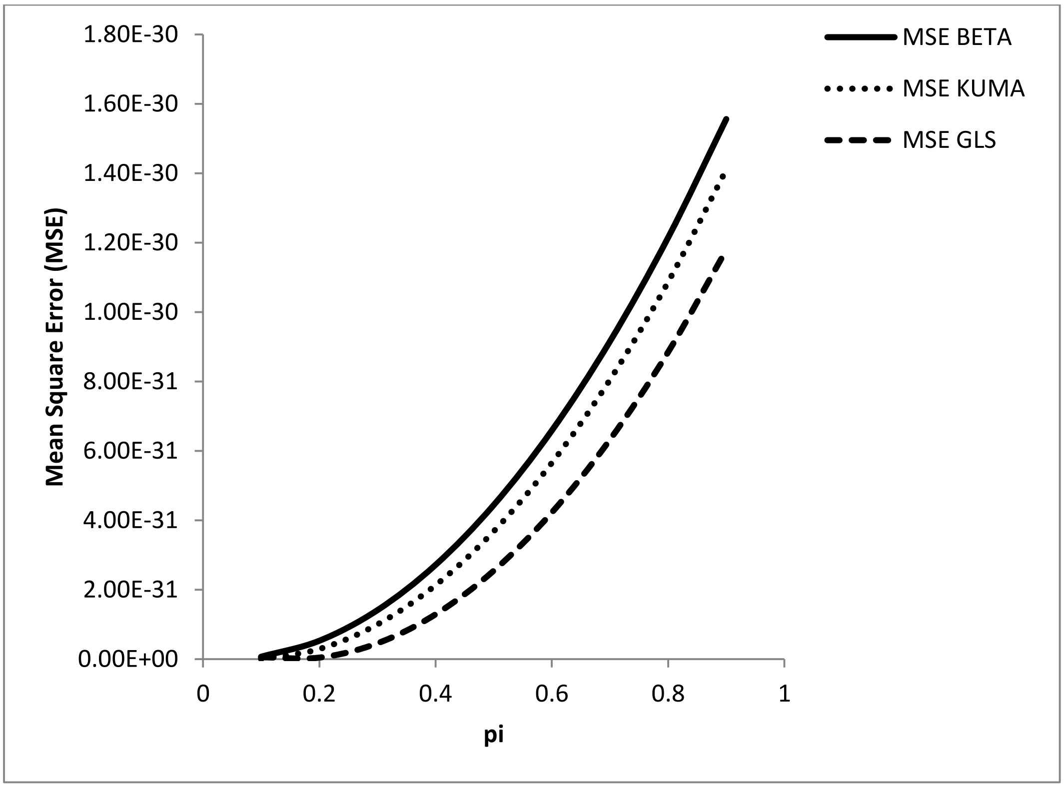

Figure 5. Mean square errors for Hussain and Shabbir (2007) RRT at
$n = 100, x = 43, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

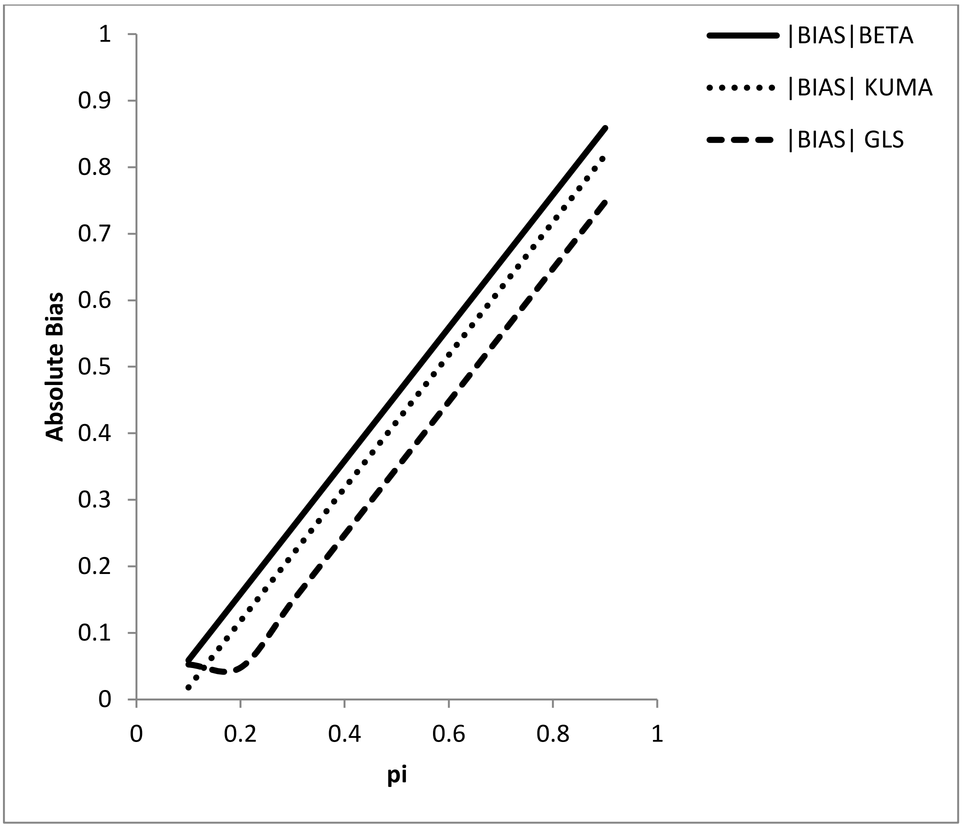

Figure 6. Absolute Bias for Hussain and Shabbir (2007) RRT at $n = 100, x = 43, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

Comment: When $n = 100, P_1 = 0.1$, the proposed estimators are better than the conventional simple beta estimator when π lies within the range $0.1 \le \pi \le 1$. However, the proposed generalised beta estimator is the best in capturing responses from respondents when π lies within the range $0.1 \le \pi \le 1$ respectively.

Table 7. Mean square errors and Relative Efficiency for Hussain and Shabbir (2007) RRT at $n = 100, x = 43, \alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$

π	MSE BETA	MSE KUMA	MSE GLS	RE KUMA	RE GLS
0.1	2.791151E-31	4.914985E-31	6.731825E-31	175.9856631	241.2186380
0.2	1.467660E-31	3.089766E-31	4.559816E-31	210.2040816	310.2040816
0.3	5.659053E-32	1.686283E-31	2.809543E-31	298.5865724	496.4664311
0.4	8.588608E-33	7.045355E-32	1.481005E-31	820.7217695	1722.933644
0.5	2.760257E-33	1.445236E-32	5.742033E-32	525.3623188	2079.710145
0.6	3.910548E-32	6.247455E-34	8.913720E-33	1.598465473	22.78772379
0.7	1.176243E-31	2.897070E-32	2.580680E-33	24.57627119	2.186440678
0.8	2.383166E-31	9.949022E-32	3.842121E-32	41.80672269	16.13445378
0.9	4.011825E-31	2.121833E-31	1.164353E-31	52.86783042	28.92768080

Table 8. Absolute Bias for Hussain and Shabbir (2007) RRT at
$n=100, x=43, \alpha=1, \beta=10, P_1=0.2, P_2=0.8$

π	|BIAS|BETA	|BIAS|KUMA	|BIAS|GLS
0.1	0.36381991	0.48278740	0.56501661
0.2	0.26381991	0.38278740	0.46501661
0.3	0.16381991	0.28278740	0.36501661
0.4	0.06381991	0.18278740	0.26501661
0.5	0.03618009	0.08278740	0.16501661
0.6	0.13618009	0.01721260	0.06501661
0.7	0.23618009	0.11721260	0.03498339
0.8	0.33618009	0.21721260	0.13498339
0.9	0.43618009	0.31721260	0.23498339

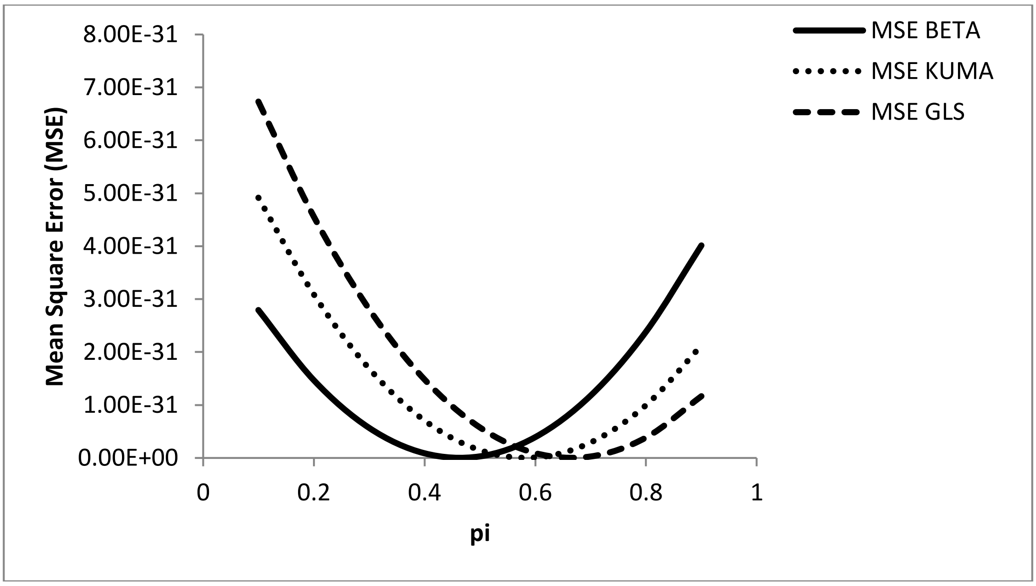

Figure 7. Mean square errors for Hussain and Shabbir (2007) RRT
$n=100, x=43, \alpha=1, \beta=10, P_1=0.2, P_2=0.8$

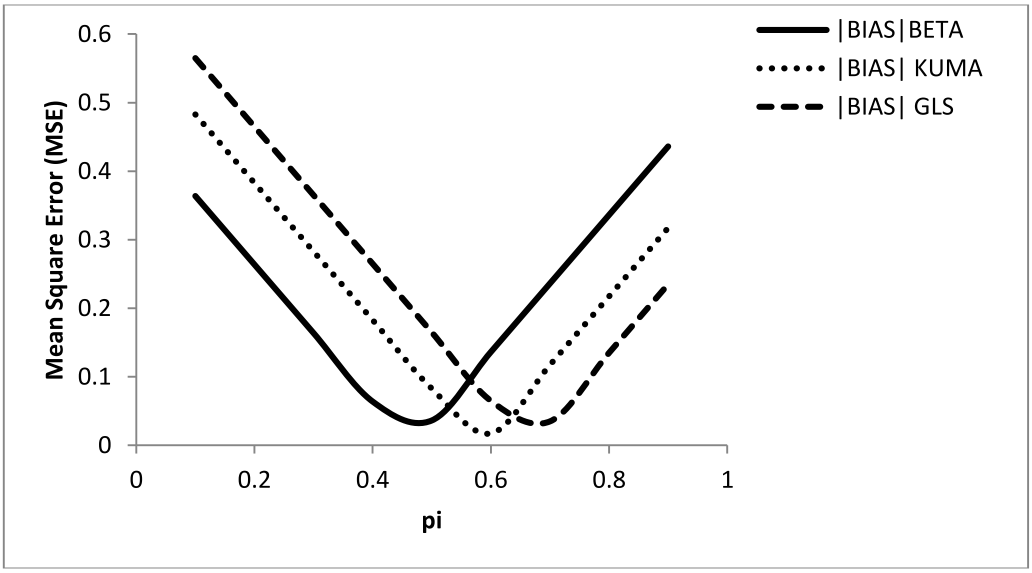

Figure 8. Absolute Bias for Hussain and Shabbir (2007) RRT at
$n=100, x=43, \alpha=1, \beta=10, P_1=0.2, P_2=0.8$

Comment: When $n = 100, P_1 = 0.2$, the conventional simple beta estimator is better than the proposed estimators when π lies within the range $0.1 \le \pi \le 0.6$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.5 \le \pi \le 1$. However, the proposed generalised beta estimator is the best in capturing responses from respondents when π lies within the range $0.6 \le \pi \le 1$ respectively.

Table 9. Mean square errors and Relative Efficiency for Hussain and Shabbir (2007) RRT at $n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

π	MSE BETA	MSE KUMA	MSE GLS	RE KUMA	RE GLS
0.1	6.762010E-77	4.132263E-77	9.538537E-78	61.09467456	14.11242604
0.2	3.327631E-76	2.705011E-76	1.717961E-76	81.38138138	51.65165165
0.3	7.986527E-76	7.004263E-76	5.348004E-76	87.60951189	66.95869837
0.4	1.465289E-75	1.331098E-75	1.098551E-75	90.47619048	74.82993197
0.5	2.332672E-75	2.162517E-75	1.863049E-75	92.70386266	79.82832618
0.6	3.400802E-75	3.194682E-75	2.828293E-75	93.82352941	83.23529412
0.7	4.669678E-75	4.427594E-75	3.994285E-75	94.86081370	85.43897216
0.8	6.139301E-75	5.861253E-75	5.361022E-75	95.43973941	87.29641694
0.9	7.809671E-75	7.495658E-75	6.928507E-75	96.03072983	88.73239437

Table 10. Absolute Bias for Hussain and Shabbir (2007) RRT at $n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

π	|BIAS|BETA	|BIAS|KUMA	|BIAS|GLS
0.1	0.08207837	0.06416302	0.03082703
0.2	0.18207837	0.16416302	0.13082703
0.3	0.28207837	0.26416302	0.23082703
0.4	0.38207837	0.36416302	0.33082703
0.5	0.48207837	0.46416302	0.43082703
0.6	0.58207837	0.56416302	0.53082703
0.7	0.68207837	0.66416302	0.63082703
0.8	0.78207837	0.76416302	0.73082703
0.9	0.88207837	0.86416302	0.83082703

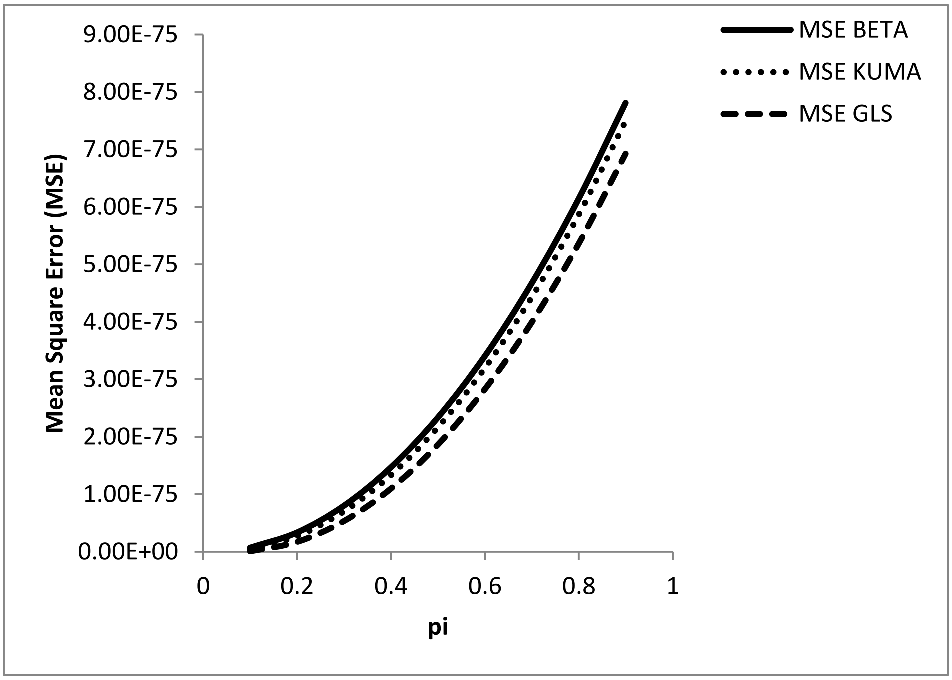

Figure 9. Mean square errors for Hussain and Shabbir (2007) RRT at
$n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

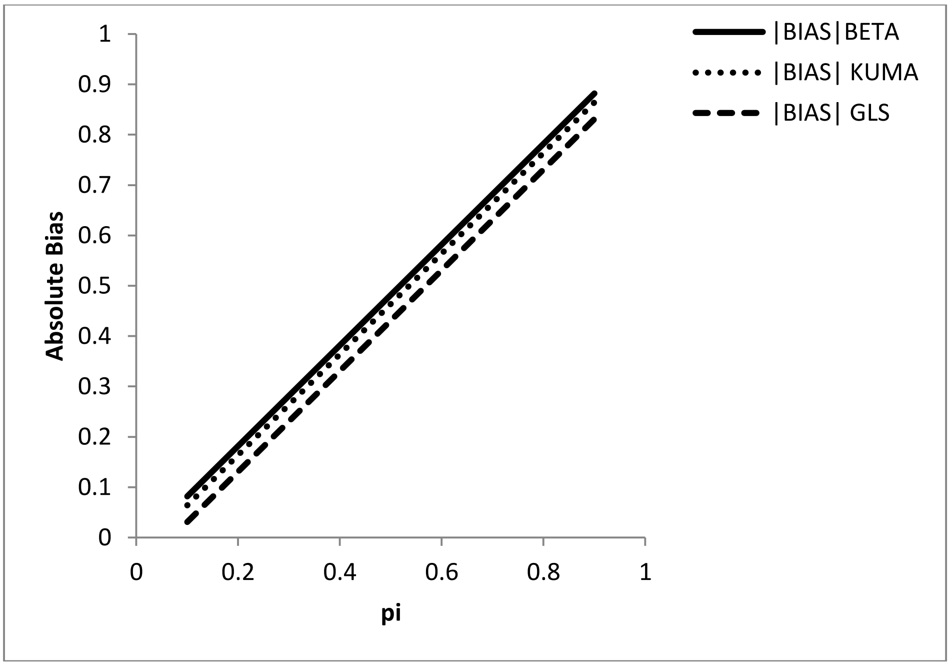

Figure 10. Absolute Bias for Hussain and Shabbir (2007) RRT at
$n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.1, P_2 = 0.9$

Comment: When $n = 250, P_1 = 0.1$, the proposed estimators are better than the conventional simple beta estimator when π lies within the range $0.1 \le \pi \le 1$. However, the proposed generalised beta estimator is the best in capturing responses from respondents when π lies within the range $0.1 \le \pi \le 1$ respectively.

Table 11. Mean square errors and Relative Efficiency for Hussain and Shabbir (2007) RRT at $n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$

π	MSE BETA	MSE KUMA	MSE GLS	RE KUMA	RE GLS
0.1	6.286780E-77	1.617733E-75	2.278710E-75	2575.516693	3624.801272
0.2	3.221153E-76	9.121848E-76	1.422586E-75	283.2298137	440.9937888
0.3	7.821096E-76	4.073836E-76	7.672089E-76	52.04603581	98.08184143
0.4	1.442850E-75	1.033292E-76	3.125784E-76	7.152777778	21.73611111
0.5	2.304338E-75	2.144654E-80	5.869454E-77	0.000930435	2.552173913
0.6	3.366572E-75	9.746041E-77	5.557399E-78	2.893175074	0.164985163
0.7	4.629553E-75	3.956461E-76	1.531669E-76	8.552915767	3.304535637
0.8	6.093281E-75	8.945784E-76	5.015232E-76	14.69622332	8.243021346
0.9	7.757756E-75	1.594257E-75	1.050626E-75	20.48969072	13.53092784

Table 12. Absolute Bias for Hussain and Shabbir (2007) RRT at $n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$

π	|BIAS|BETA	|BIAS|KUMA	|BIAS|GLS
0.1	0.07914162	0.401461737	0.47646975
0.2	0.17914162	0.301461737	0.37646975
0.3	0.27914162	0.201461737	0.27646975
0.4	0.37914162	0.101461737	0.17646975
0.5	0.47914162	0.001461737	0.07646975
0.6	0.57914162	0.098538263	0.02353025
0.7	0.67914162	0.198538263	0.12353025
0.8	0.77914162	0.298538263	0.22353025
0.9	0.87914162	0.398538263	0.32353025

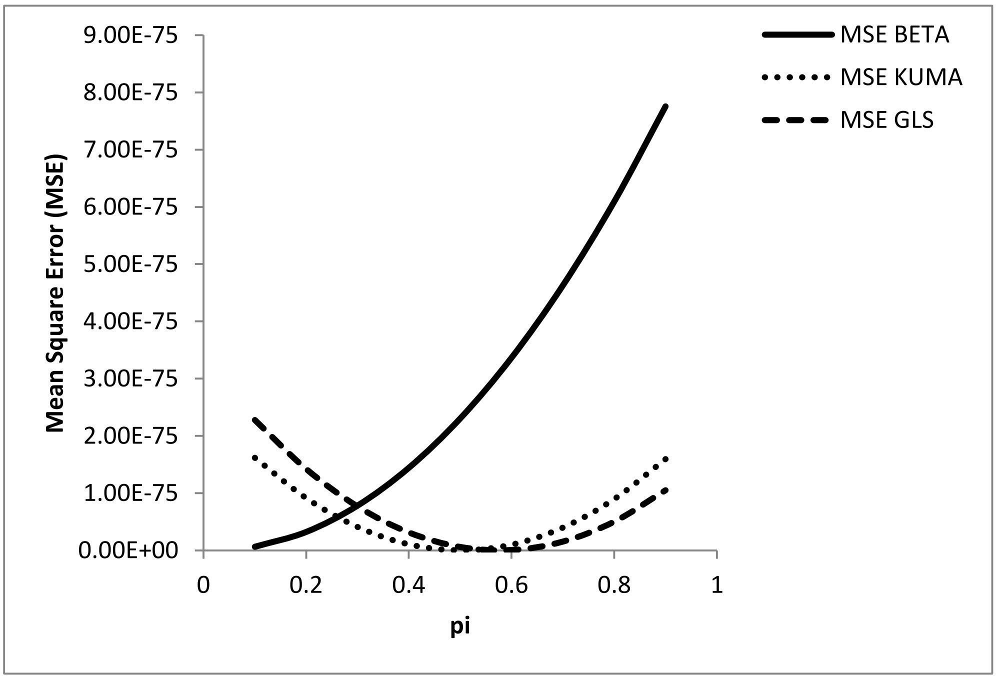

Figure 11. Mean square errors for Hussain and Shabbir (2007) RRT at $n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$

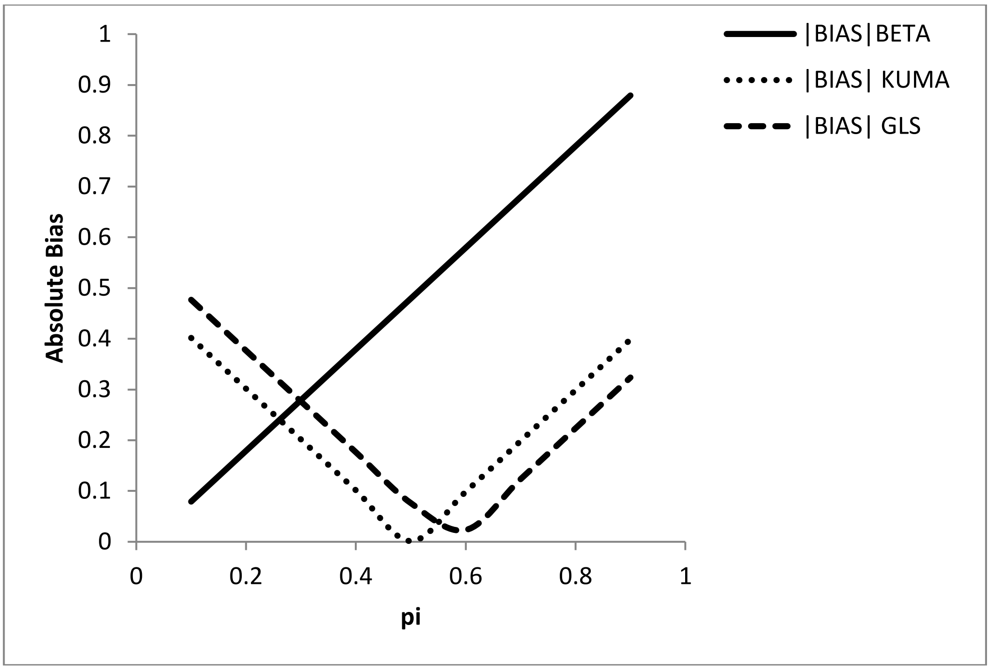

Figure 12. Absolute Bias for Hussain and Shabbir (2007) RRT at $n = 250, x = 106, \alpha = 1, \beta = 10, P_1 = 0.2, P_2 = 0.8$

Comment: When $n = 250, P_1 = 0.2$, the conventional simple beta estimator is better than the proposed estimators when π lies within the range $0.1 \le \pi \le 0.3$ while the proposed estimators are better than the conventional estimator when π lies within the range $0.2 \le \pi \le 1$. However, the proposed generalised beta estimator is the best in capturing higher responses from respondents when π lies within the range $0.5 \le \pi \le 1$ respectively.

Results and Discussion

From the results presented in Table and Fig. 1 to 12 respectively, when $n = 25, P_1 = 0.1$, the proposed Bayesian estimators are better than the conventional simple beta estimator when π lies within the range $0.4 \le \pi \le 1$. However, the proposed generalised beta estimator is the best in capturing responses from respondents when π lies within the range $0.7 \le \pi \le 1$.

When $n = 25, P_1 = 0.2$, the proposed Bayesian estimators are better than the conventional simple beta estimator when π lies within the range $0.5 \le \pi \le 1$. However, the proposed generalised beta estimator is the best in capturing responses from respondents when π lies within the range $0.7 \le \pi \le 1$.

When $n = 100, 250, P_1 = 0.1$, the proposed estimators are better than the conventional simple beta estimator when π lies within the range $0.1 \le \pi \le 1$. However, the proposed generalised beta estimator is the best in capturing responses from respondents when π lies within the range $0.1 \le \pi \le 1$.

When $n = 100, P_1 = 0.2$, the proposed Bayesian estimators are better than the conventional simple beta estimator when π lies within the range $0.5 \le \pi \le 1$. However, the proposed generalised beta estimator is the best in capturing responses from respondents when π lies within the range $0.6 \le \pi \le 1$.

When $n = 250, P_1 = 0.2$, the proposed estimators are better than the conventional simple beta estimator when π lies within the range $0.2 \le \pi \le 1$. However, the proposed generalised beta estimator is the best in capturing responses from respondents when π lies within the range $0.5 \le \pi \le 1$ respectively.

Conclusion

We have developed the alternative Bayesian estimators of the population proportion of respondents with respect to stigmatized attribute when life data were gathered through the administration of questionnaires on abortion on 300 matured women in some selected hospitals in Akure, Ondo State assuming both Kumaraswamy (KUMA) and the generalised (GLS) beta priors as our alternative beta prior distributions in addition to simple beta prior distribution used by Hussain and Shabbir (2009a). We observed clearly from the results presented in tables and figures above, that for small, intermediate as well as large sample sizes, the proposed Bayesian estimators were more sensitive in capturing responses from respondents than that of Hussain and Shabbir (2009a). In particular, the proposed generalised beta estimator is the best in capturing information from respondents in survey which asks sensitive questions.

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