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MINIMIZING COST OF TRANSPORTATION BY VOGEL'S APPROXIMATION METHOD: A CASE STUDY OF MAISANGO CEMENT NIGERIA PLC

Ibrahim Abdullahi[a], Rabiu Hamisu Kankarofi[b], Usman Abbas Yakubu[c,✉], Abba V Mandara[d], Salisu Murtala[e]

[a,b,c]Department of Mathematics, Yusuf Maitama Sule University, Kano, Nigeria.
[d]Department of Mathematics, University of Maiduguri, Borno, Nigeria.
[e]Department of Mathematics, Federal University Dutse, Jigawa, Nigeria.
✉usman.abbas84@yahoo.com

Abstract: This piece of research was conducted to optimize the procedures of transporting and distributing of Cement products from MAISANGO Cement Plc to the require destinations which resultant to the reduction of operating cost of transportation problems that usually occurs due to the nature of bad roads to some selected locations in Lagos state- Nigeria. To minimize the cost, a mathematical optimization model (Vogel Approximation method) has been utilized to ease the processes of transporting the product to the various locations.

Keywords: Vogel Approximation method; Optimal solution; Least Cost method; Northwest Corner method.

1. INTRODUCTION

Transportation model was originally formulated by Kantorovich (1942). However, the model could solve the transportation problems optimally for complex business problems. The transportation problem can be described using Linear Programming Mathematical model and usually appears in a transportation tableau. The initial basic feasible solution of any transportation problem is obtained by well-known methods including Northwest Corner method, Row Minimization method or Least-Cost method [14]. In this work the optimality of the given transportation problem is certified by Vogel Approximation method.

The transportation models are basically involves m sources, each of which has available S_i where $i = 1, 2, 3, 4, \dots, m$ units of a homogeneous products and n dimensions, each of which requires d_j where $j = 1, 2, 3, 4, \dots, n$ units. The number S_i and d_j are always positive integers. The transporting cost C_{ij} of one unit of product from the source i^{th} to the j^{th} location is given for each i and j.

To develop an integral transportation schedule that meets all demands from current inventory at minimum total distribution cost. It is assumed that total supply and total demand are definitely equal, i.e.

$$\sum_{j=1}^{n} d_j = \sum_{i=1}^{m} S_i \qquad (1)$$

The expression (1) is certain by creating either a fictitious destination with a demand equal to the supply.

2. DETAILS DESCRIPTION OF THE METHODS

Mathematical models have been utilized to minimize the cost of transportation. [2] the application of the transportation problems tends to have a very large number of constraints and variables, so a straight forward computer application of the simplex method may require an excessive computation effort, but in 1951, [1] applied the concept of Linear Programming in solving the transportation [13,2,5,3,7,8,10,11,4,6,9]. [1] gave the standard linear programming formulation of transportation problem and applied the simplex method to find the solution. Since then the transportation problem has the classical common subject in almost every text.

This presentation is considered to be the first important contribution to the solution of transportation problem. [9], presented an independent study, not related to [12], and called it "optimum utilization of the transportation system". The two contributions aided in the development of transportation methods which involves a number of sources and its destinations. [12] developed an algorithm for reaching an optimal solution of the transportation problems for the convex case. Fortunately, a key characteristic is most of the X_{ij} coefficient in the constraints are zeros and relatively few none-zero coefficients appear in a distinctive pattern [12,16,15,17,18].

The general model for the transportation problem from source to the destinations in order to minimize the total cost, i.e. sources $i = 1,2,3,\ldots,m$ has demanded for d_j to be received from the source i to the destination j this process is directly proportional to the number distributed, where C_{ij} denoted the cost per unit distribution, and letter Z to be the total distribution cost and X_{ij} for $i = 1,2,3,4,\ldots,m,\ and\ j = 1,2,3,4,\ldots,n$ to be the number of units to be distributed from source i to destination j. Each unit transportation cost function is a linear in nature, the linear programming problem is given as follows: -

$$minimize = \sum_{j=1}^{n}\sum_{i=1}^{m} C_{ij}\, X_{ij} \tag{2}$$

$$subject\ to \sum_{j=1}^{n} X_{ij} = S_i \qquad for\ j = 1,2,3,\ldots,m$$

$$and \sum_{i=1}^{m} X_{ij} = d_j \qquad for\ i = 1,2,3,\ldots,n$$

$for\ all\ X_{ij} > 0,$ for all i and j.

The transportation problems arise in planning for the distributions of goods and services from different supply locations to different destinations.

3. DATA ANALYSIS

The figures were collected from the marketing department of MAISANGO Cement Nigeria Plc. The request on the cement product to various locations in Lagos state Nigeria are always very high. This research will focus on five (5) towns in Lagos state that comprises **Ikeja, Ajagunle Apapa, Idi-Araba and Agege town**. The information attained are summarized and presented in table 2 and table 3 below. The general Transportation problem are itemised by the following information:

- A set of m supply points from which a product is shipped. Supply point i can supply at most S_i units.
- A set of n demand points to which the product is shipped. Demand point j must receive at least d_j units of the shipped product.
- Each unit product at supply point i and shipped to demand point j gives available cost of C_{ij}.

Therefore, if the total demand is less than total supply, or a fictions source with a supply equal to the shortage, or the total demand exceeds total supply. Then, the set of constraints

$$\sum_{j=1}^{n} X_{ij} = S_i\ and \sum_{i=1}^{m} X_{ij} = d_j$$

Then $S_i = d_j$ the transportation problem is called balanced problem.

$$if \left(\sum_{j=1}^{n} X_{ij} = S_i\right) \neq \left(\sum_{i=1}^{m} X_{ij} = d_j,\right)$$

then the transportation problem is called unbalanced transportation problem. And if it is unbalanced, then there are two possible cases:

Case (1)

$$if \left(\sum_{j=1}^{n} X_{ij} = S_i \right) > \left(\sum_{i=1}^{m} X_{ij} = d_j , \right)$$

then, introduce dummy row with zeros as cost to balance it.

Case (2)

$$if \left(\sum_{j=1}^{n} X_{ij} = S_i \right) < \left(\sum_{i=1}^{m} X_{ij} = d_j \right),$$

then introduce dummy column with zeros as cost to balance the transportation problem. Consequently, the relevant data can be formulated in a table 1 and flow chart of transportation problem below:

Flow Chart of Transportation Problem

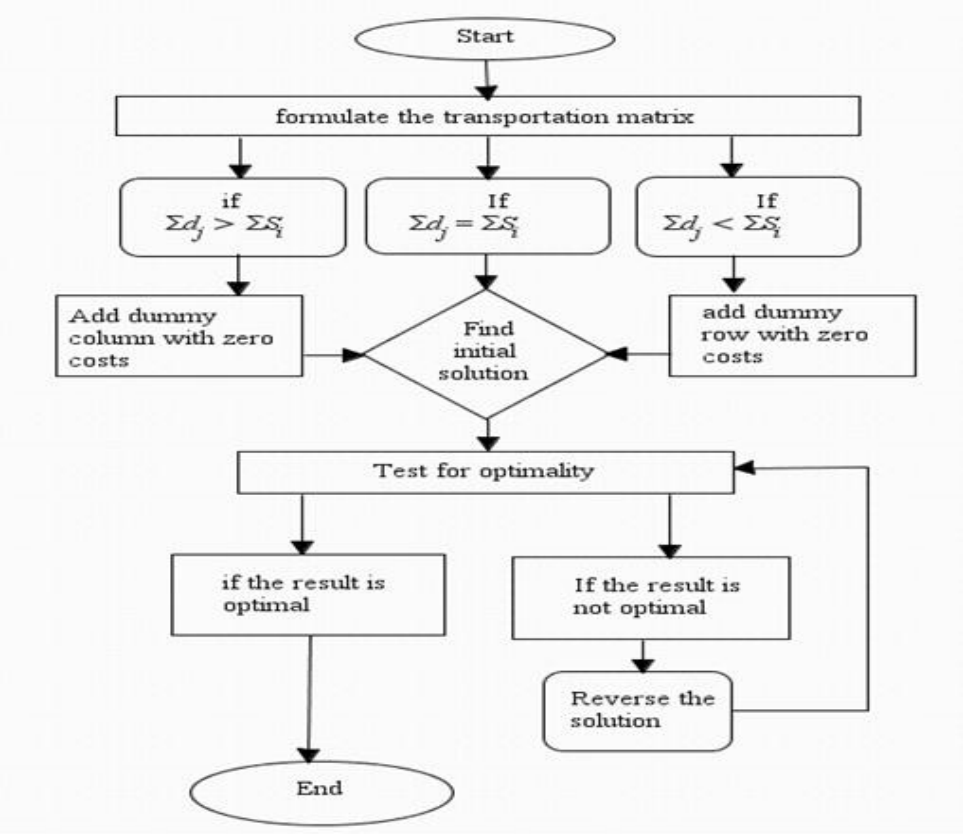
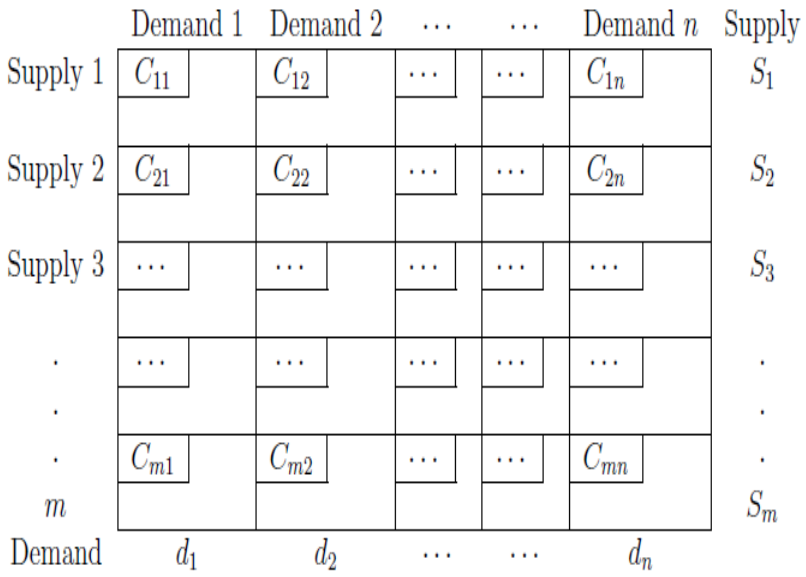


Table 1. Transportation Tableau



4. DATA PRESENTATION

MAISANGO Cement Nigeria Plc has four cement depots in Lagos state the total bags of Cement supplied from the company and total bags demanded for the respective locations in Lagos state are Apapa, Ikeja, Ajagunle, Idi-Araba and Agege. Table 1 shows the total bags of Cement demanded in each location given as follows:

Table 2. Bags of Cement Demanded by Five Towns in Lagos State

	Ikeja	**Ajagunle**	**Apapa**	**Idi-Araba**	**Agege**	**Total Supply**
Depot (A)	150	200	180	370	200	1100
Depot (B)	210	120	400	300	190	1220
Depot (C)	250	250	300	100	50	950
Depot (D)	400	400	100	480	450	1830
Total Demand	1010	970	980	1250	890	

The cost of sending one bag of Cement from depots to the destinations depends on the distance the truck must travel. Below is the table of cost of transporting Cement from depots to their respective towns in Lagos state in Nigerian currency.

Table 3. Cost of Transportation Per Bag in Naira (₦)

	Ikeja	**Ajagunle**	**Apapa**	**Idi-Araba**	**Agege**
Depot (A)	68	36	26	31	62
Depot (B)	35	72	36	28	54
Depot (C)	40	55	41	30	28
Depot (D)	25	35	40	80	34

We utilized equation (2) above subject to the condition bound on it and for all X_{ij} non-negative, for all i and j. We are determining to use the four methods listed to find out the best among them in terms of number of iterations in order to achieve a required result.

Northwest Corner Method

Commencing by the upper left corner of the transportation tableau and set X_{11} as large as possible, clearly, X_{11} can be no larger than the smaller of S_i and d_j.

- If $X_{11} = S_1$ cross out the first row of the tableau. Also change d_1 to $d_1 - S_1$.
- If $X_{11} = d_1$ cross the first column of the tableau. Change S_1 to $S_1 - d_1$.
- If $X_{11} = S_1 = d_1$ cross out either row 1 or column 1 but not both.
    - If you cross out row, change d_1 to 0.
    - If you cross out column, change S_1 to 0.

Continue applying this procedure to the most Northwest cells in the tableau that does not lie in a crossed out row or column. Eventually, you will come to a point where there is only one cell that can be assigned a value. Assign this cell a value equal to its row or column demand, and cross out both the cell's row or column, a basic formulating solution now been obtained.

Table 4. Northwest Corner Method

	Ikeja	Ajagunle	Apapa	Idi-araba	Agege	Supply	U_i
Depot (A)	68 1010	36 90	26	31	62	1100	U_1
Depot (B)	35	72 880	36 340	28	54	1220	U_2
Depot (C)	40	55	41 640	30 310	28	950	U_3
Depot (D)	25	35	40	80 940	34 890	1830	U_4
Demand	1010	970	980	1250	890		
V_j	V_1	V_2	V_3	V_4	V_5		

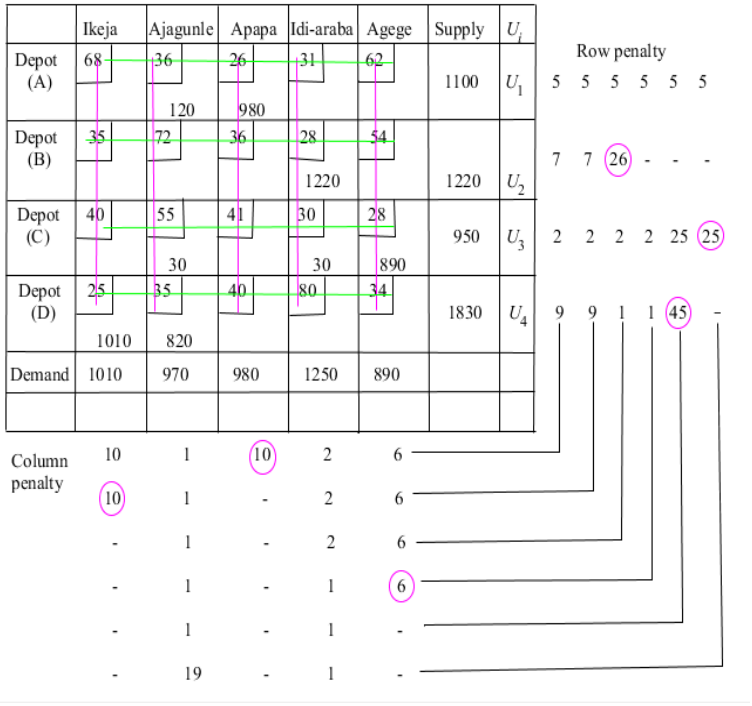
$$Minimize\ (Z) = (68 \times 1010) + (36 \times 90) + (72 \times 880) + (36 \times 340) + (41 \times 640) + (30 \times 310) + (80 \times 940) + (34 \times 890) = 288{,}520$$

Vogel Approximation Method

Computing each row and column a penalty equal to the difference between the two smallest costs in the row and column, next find the row or column with the largest penalty, choose as the first basic variable, i.e. the variable in this row or column that has the smallest costs. As describe in the Northwest corner Method, make this variable as large as possibly, cross out row or column, and change the supply or demand associated with the basic variable.

Now re-compute new penalties (using only cells that do not lie in a crossed out row or column), and repeat the procedure until only one uncrossed cell remains. Set this variable equal to the supply or do demand associates with the variable and cross out the variable's row and column Goyal (1984).

Table 5. Vogel Approximation Method



Where penalty is equal to second to the least C_{ij} – least C_{ij} (for both row and column), after finding penalties between rows and column, then, select the highest one and find the smallest $\boldsymbol{C_{ij}}$ then assign order to it by considering the supply and demand of that row or column. When the orders of row or column was exhausted, cancel that row or column and not used any $\boldsymbol{C_{ij}}$ from it, then continue by applying the same procedure until all the rows and column are balanced.

$$Minimize\ (Z) = (36 \times 120) + (26 \times 980) + (28 \times 1220) + (55 \times 30) + (30 \times 30) + (28 \times 890) + (25 \times 1010) + (35 \times 820) = 145{,}380$$

Least Cost Method

The least cost method is more economical compared to other methods, since it begins with a lowest cost in the transportation table. The following steps is used to

- Find the cell with the lowest cost in the table.
- Assign maximum feasible value to this cell.
- Remove the row or column where an assignment is made.
- Repeat the procedure above to reduce the table until all assignments are made.

Table 6. Least Cost Method

	Ikeja	Ajagunle	Apapa	Idi-araba	Agege	Supply	U_i
Depot (A)	68	36 970	26 130	31	62	1100	U_1
Depot (B)	35	72	36 850	28 370	54	1220	U_2
Depot (C)	40	55	41	30 880	28 70	950	U_3
Depot (D)	25 1010	35	40	80	34 820	1830	U_4
Demand	1010	970	980	1250	890		
V_j	V_1	V_2	V_3	V_4	V_5		

$$Minimize\ (Z) = (36 \times 970) + (26 \times 130) + (36 \times 850) + (28 \times 370) + (30 \times 880) + (28 \times 70) + (25 \times 1010) + (34 \times 820) = 160{,}750$$

Row Minima Method

In the row minima method,

- Assign as much as you can in the minimum cost cell of the first row i.e. min (S_i, d_j).
- Subtract this least value from supply S_i and demand d_j. Therefore, if the supply is zero, then strike that row and if the demand is zero then cross that column, and if minimum cost cell is different, then choose the cell where maximum allocation will be possible.
- Repeat this procedure for all rows and columns that is not strike before until all demand and supply values equal to zero.

Table 7. Row Minima Method

	Ikeja	Ajagunle	Apapa	Idi-araba	Agege	Supply	U_i
Depot (A)	68	36	26 980	31 120	62	1100	U_1
Depot (B)	35 90	72	36	28 1130	54	1220	U_2
Depot (C)	40 60	55	41	30	28 890	950	U_3
Depot (D)	25 860	35 970	40	80	34	1830	U_4
Demand	1010	970	980	1250	890		
V_j	V_1	V_2	V_3	V_4	V_5		

$$Minimize\ (Z) = (26 \times 980) + (31 \times 120) + (35 \times 90) + (28 \times 1130) + (40 \times 60) + (28 \times 890) + (25 \times 860) + (35 \times 970) = 146{,}760$$

Basic Feasible Solution

When solving transportation problem, the initial basic feasible solution (IBFS) cannot be considered as final solution, because may be there exist another solution which can reduce IBFS after some iteration, if that solution is reduce, then it is called basic feasible solution (BFS). The optimality test must be used to evaluate the solution if there is another technique of improving the BFS. Without using any mathematical model i.e., the initial cost, the company should spend ₦217,150 as transportation cost, but after using mathematical method the amount reduced to ₦145,140. This is shows that there is discount of ₦72,010. Northwest corner method, Vogel approximation method, Least Cost method and Row minima method has been used to minimize the cost of transportation.

5. RESULTS AND DISCUSSION

In this section, we clearly discuss the cost of transportation of cement product from the MAISANGO Nigeria Plc to the required destinations in Lagos Nigeria. From section 3 above the results shows that when finding the initial basic feasible solution IBFS, all the four methods presented gives different solution. The IBFS of Northwest corner method gives ₦288,520, Least cost method gives ₦146,750 and Row minima method gives ₦146,760 and the Vogel approximation method gives ₦145,380 which is the lowest cost of transportation from the sources to the destinations compared to other methods used in this research. This clearly shows that Vogel approximation method is the most efficient among the remaining three methods in finding the initial basic feasible solution, because it has the least amount as transportation cost and less number of iterations before optimality test. A feasible solution (not necessarily basic) is said to be Optimal if it minimizes the total transportation cost.

Table 8. Optimality Solution

Tested Method	IBFS	Number of Iteration	Optimal Solution
Northwest Corner Method	₦288,520	**9**	₦145,140
Vogel Approximation Method	₦145,380	**2**	₦145,140
Least Cost Method	₦160,750	**3**	₦145,140
Row Minima Method	₦146,760	**4**	₦145,140

Note that all the methods give the same result, and this result show that it reached the minimum amount for transportation of the product. The result shows that Northwest corner method have reached basic feasible solution after 9 iterations which cost ₦145,140, Least Cost method cost ₦145,140 after 3 iterations and Row minima reduces the amount to ₦145,140 after 4 iterations while by using Vogel approximation method the cost of transportation reached ₦145,140 after 2 iterations. This transportation model will benefit the company in making optimum allocations of its product from the four depots to their various customers in different destinations in and outside Lagos state at a minimum transportation cost.

6. CONCLUSION

This research was carried out in order to attain the minimum cost of transporting cement product from sources to the destinations in Lagos Nigeria. The Company was able to know how much they spend to supply their customers for them to minimize the total cost of transportation. Vogel approximation method was utilized and found to be efficient and firm towards the profitability of the company.

Conflicts of Interest

There is no any conflict of interest regarding the publication of this article.

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