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CALCULATING CONFIDENCE INTERVAL ESTIMATION USING MAXIMUM LIKELIHOOD ESTIMATOR FOR DATASET

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Abstract: Confidence interval estimation is an important technique in statistical inference. The objective of this paper is to demonstrate the calculation of confidence interval using maximum likelihood method for surface roughness. The advantages of maximum likelihood method are convenience to use and computational efficient. The interval length of confidence interval showed that 99% is the highest width compared to 90% and 95%.

Keywords: *Confidence Interval, Maximum Likelihood estimation.*

1. INTRODUCTION

There is a different type of sampling in statistics. The random sampling method guarantees that every individual in the population has an equal probability of being selected. Random sampling allows probability theory concepts to be employed to data (Hazra, 2017). The data from samples can be described and draw inferences. A statistical population is a group of entities from whom statistical inferences are to be formed, usually from a random sampling of the population.

Confidence interval estimation plays an important part in statistical inferences about a population parameter and the accuracy of its estimator. Modelers can utilise confidence interval estimation to build their confidence in parameter estimates (Dogan, 2004). Study by Tan and Tan (2010) showed that confidence intervals are a key output of numerous statistical analyses and serve an important part in the estimate's interpretation of parameters. An interval of values generated from sample data that is plausible to encompass the real parameter of interest is called a confidence interval. The difficulty of determining the confidence intervals around the best-fit parameter is at a minimum as challenging as obtaining the best estimates themselves due to violations of standard assumptions, for instance, identically and independently distributed (iid) normal error terms. The confidence interval was created to check if the result differed considerably from zero or any other value.

According to Fethney (2010), confidence intervals are a tool that can assist a researcher in determining values in a larger population. Confidence intervals comprise a range of possible values with upper and lower boundaries that include the unidentified population. Note that 90%, 95%, or 99% confidence intervals are commonly employed. When 95% confidence intervals are estimated, there is a 95% chance that the interval, which is bordered by the upper and lower limits, encompasses the 'true' population value, or parameter, having a 5% probability that the interval does not.

According to Gábor & Banga (2015), the goal of parameter estimate is to determine the unknown model parameters that best suit a set of experimental data. Estimating system dynamics models presents a variety of issues. Hence, many conventional statistical methods do not directly apply (Hosseinichimeh, Rahmandad, Jalali, & Wittenborn; 2016). Failure to define unknown parameters during the design phase might have an impact on dynamic behaviour, resulting in a lack of relevant insight, accurate forecasts, robust control, and acceptable designs for future experiments (Matthias Chung, Mickaël Binois, Robert, Gramacy, 2019).

Parameter estimation aims to identify the unknown parameters of the model giving the best fit to a set of experimental data ayat seperti berulang. Dynamic modelers are frequently confronted with difficulties requiring the estimation of model parameters from numerical empirical data. In many filed of science and engineering, the challenge of parameter estimation in system dynamics arises. The model's shape can often be generated from previous knowledge of the process under examination; however, the model's parameters should be derived from empirical data in the time series data form. Incomplete measures have been developed in a broad range of disciplines, which include state space modeling in econometrics and statistics, control theory in engineering, nonlinear dynamics in physics, as well as ergodic theory and dynamical systems in mathematics since this issue has emerged in a range of circumstances. The goal of this study is to illustrate how to use the maximum likelihood method to calculate confidence intervals.

In the first section is the introduction. Followed by section 2 presenting methods and material. Then section 3 offers the discussion and results. Lastly, section 4 discusses the paper's conclusion.

2. MATERIAL AND METHODS

This section provides brief about the concept of point estimates and confidence intervals. Inferential statistics is a set of tools and approaches that enables us to derive inferences regarding populations using data from a sample.

2.1 Point estimates

Single numeric numbers are created from sample data to offer a statistical approximation to population parameters of interest, which is known as a point estimate (Lavrakas, 2008). Assume that surveys were planned to estimate the relevant population quantities: (a) workers' monthly average salary, (b) primary school pupils' average weight, as well as (c) the total amount of text messages sent by mobile phone subscribers of a certain mobile phone carrier in the preceding month. As a result, data from a subset or sample of the population is gathered, the sample mean statistic is calculated, and the sample mean statistic is viewed as an estimate of the population mean parameter. The population mean, μ, is estimated by utilising a sample mean, $\overline{x}$.

2.2 Confidence intervals

Instead of a single number, interval estimates try to estimate a parameter utilizing a range of values (Lavrakas, 2008). An unknown population parameter's interval estimate generated from a sample chosen from that population, which there is a known, and statistically justifiable degree of confidence that the unknown population parameter lies within that interval is described as a confidence interval (Petty, 2012). A two-sided confidence interval (CI) separates the population parameter from the upper and lower boundaries, as per Hazra (2017). Jerzy Neyman established the notion of confidence intervals in a research presented in 1973. A confidence level is utilized to construct interval estimates, which is the probability that the interval reflects the population parameter being estimated.

2.3 Meaning of confidence interval

Even though confidence intervals offer an estimate of an unknown population parameter, the interval computed from a particular sample does not always reflect the true value of the parameter. Note that confidence intervals are calculated at a given level of confidence. This indicates that 95% of the confidence intervals should encompass the true value if the estimate procedure is repeated numerous times utilizing random samples from the identical population. It is worth noting that the reported level of confidence is chosen by the user and is unaffected by the sample's characteristics. Despite 95% confidence intervals are certainly the most common, confidence intervals can be calculated at any confidence level, for instance, 90% or 99%. The population parameter is bracketed by a two-sided confidence interval that spans the lower and upper boundaries (Hazra, 2017).

3. RESULTS AND DISCUSSION

Illustrative example:

Surface roughness is assessed using fatigue test data (Zhang, Liu, Wang, & Wang; 2019). The 13 data on the roughness surface are 3.1, 3.1, 3.2, 3.2, 3.2, 3.3, 3.3, 3.3, 3.4, 3.4, 3.4, 3.5, and 3.5. The maximum likelihood estimator approach is utilized to estimate the parameters of the original test data.

General formulas for calculating confidence interval for system dynamic using maximum likelihood estimator.

Confidence interval = Point estimate ± Margin of error
= Point estimate ± Critical value (t) x Standard error of point estimate
= Point estimate ± Critical value (t) x Standard deviation /$\sqrt{n}$

The statistic produced from sample data is referred to as a point estimate. The critical value, also known as the *t* value, is obtained from the standard normal curve's mathematics and is dependent on the confidence level. The sample standard deviations may be utilized to estimate the population standard deviation if the sample size is low. The t values are 1.7823, 2.1788, and 2.681, respectively, with confidence levels of 90%, 95%, and 99%. The sample size and distribution of the dependent variable define the standard error. As per Tan and Tan (2010), 90% confidence interval, 95% confidence interval, and 99% confidence interval may all be estimated for a particular study, having the width of the 90% confidence interval being narrower than the width of the 99% confidence interval.

Referring to the fatigue test data by Zhang, Liu, Wang, & Wang (2019), the following calculation was demonstrated to find confidence interval for 90%, 95% and 99% confidence levels.

$$\hat{\mu} = \frac{3.4 + 3.3 + 3.2 + 3.1 + 3.1 + 3.2 + 3.2 + 3.3 + 3.3 + 3.4 + 3.4 + 3.5 + 3.5}{13} = 3.3$$

$$s = \sqrt{\frac{1}{13-1}\left[141.79 - \frac{42.9^2}{13}\right]} = 0.1354$$

The original sample's point estimation is estimated to be $\hat{\mu} = 3.3$ while standard deviation is $s = 0.1354$.

a) 90% confidence level

Confidence interval = 3.3 ± 1.7823 x $\frac{0.1354}{\sqrt{13}}$
= 3.3 ± 1.7823 x 0.03755
= [3.2331, 3.3670]

The 90% confidence interval is [3.2331, 3.3670].

b) 95% confidence level

Confidence interval = 3.3 ± 2.17881 x $\frac{0.1354}{\sqrt{13}}$
= 3.3 ± 2.17881 x 0.03755
= [3.2182, 3.3818]

The 95% confidence interval is [3.2182, 3.3818].

c) 99% confidence level

Confidence interval = 3.3 ± 2.681 x $\frac{0.1354}{\sqrt{13}}$

= 3.3 ± 2.681 x 0.03755
= [3.1993, 3.4007]

The 99% confidence interval is [3.1993, 3.4007].

Table 1 summaries that 90% confidence that the average is between 3.2331 and 3.3670, 95% confidence that the average is between 3.21818 and 3.38182 and 99% confidence that the average is between 3.1993 and 3.4007.

Table 1: Critical (*t*) values employed in the confidence intervals calculation.

Confidence level	Confidence interval	Interval length
90%	[3.2331,3.3670]	0.1339
95%	[3.2182,3.3818]	0.1636
99%	[3.1993, 3.4007]	0.2014

4. CONCLUSION

The maximum likelihood technique in calculating confidence intervals for fatigue test data for surface roughness was used in this paper. Since the confidence interval for the difference scores excludes zero, the score differs substantially. The maximum likelihood technique is a well-known inference theory that is simple to compute with Vensim software. For system dynamics simulation models, a statistical distributions' estimation for ambiguous parameters is important.

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